The Planck – Kleinert Crystal

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The Planck – Kleinert Crystal hypothesis is analyzed for an ideal cubic fcc crystal formed by Planck particles. In this type of a quasi-continuum the energy, momentum, and mass transport are described by the classical balance equations. The transverse wave is the electromagnetic wave, and its velocity equals the velocity of light. The quasi-stationary collective movement of mass in the crystal is equivalent to the *particle* (body), and such an approach enables derivation of the Schrödinger equation. The diffusing interstitial Planck particles create a gravity field, and the computed value of G is within the accuracy of experimental data. The model predicts four different force fields and a vast amount of the "dark matter and dark energy" in the crystal lattice. It allows for a self-consistent interpretation of multiscale phenomena.

Key words: Planck Scale Physics; Gravity; Dark Matter; Maxwell Equations.

1. Introduction

Physics Today was opened in 1999 by F. Wilczek's article "The persistence of ether", concerned with speculations on the physical vacuum and continuum, named ether [1]. Arguments supporting the continuum concept are provided by statistical mechanics [2]. When the ratio between macro- and microscopic scales becomes very large, collective properties emerge that are not noticeable in the behaviour of individual particles. To some degree, quantum space can be considered as analogue to a crystal which, through its atomic structure, changes the propagation of light relative to that through a vacuum [3]. Already in 1987 Kleinert proposed a simple three-dimensional Euclidean World Crystal model [4]. He successfully recovered Einstein's gravity from a defect model of a crystal in which some elastic terms vanish. Recently, Kleinert and Zaanen explained the absence of torsion in space-time [5].

In the present work the World Crystal is a three-dimensional quasi-continuum, $x \in \mathbb{R}^3$, that will be called the Planck – Kleinert Crystal (PKC). The PKC is a cubic fcc crystal showing the Frenkel disorder. The proposed model is based on: (i) the solid solution assumption, (ii) the ability of defects to diffuse, (iii) the classical conservation laws connecting the mechanic and thermodynamic laws, (iv) the postulate that gravity is the result of diffusing defects (diffusing Planck particles), (v) the deformation and velocity vanish when defining the collective Planck mass movement, and (vi) the assumption that the electric field strength equals the negative of the rotation of the displacement vector. The properties of the crystal are shown in Table 1.

2. The Planck – Kleinert Crystal

The building blocks of a PKC are Planck particles, P, that obey the laws of mass, momentum and energy conservation. Each particle exerts a short range force at the Planck length, l_P. The crystal is an ideal (central forces act between particles) cubic fcc crystal showing the Frenkel disorder. The PKC is *mechanically simple*, i. e., its mechanical properties do not depend on strain, Table 1.

2.1. Defects in the PKC

Defects are treated as an ideal solid solution. The process one can consider is the formation of intrinsic Frenkel defects: $\mathrm{P_P} \rightleftarrows \mathrm{V_P} + \mathrm{P_i}$, where $\mathrm{P_P}$, $\mathrm{V_P}$ and $\mathrm{P_i}$ denote a particle in lattice position, vacancy and interstitial particle, respectively. The densities of lattice sites ($\rho = [\mathrm{P_P}] + [\mathrm{V_P}]$) and defects ($\rho_\mathrm{P} = [\mathrm{P_i}] > [\mathrm{V_P}]$) differ by orders of magnitude, $\rho \cong 4 l_\mathrm{P}^{-3} = \mathrm{const} \gg \rho_\mathrm{P}$, where $\rho_\mathrm{P} = N_\mathrm{P}\rho$ and N_P is the concentration ratio of

Table 1. The physical constants of the Planck – Kleinert Crystal (ideal regular fcc).

Physical quantity	Unit	Symbol for unit	Value in SI units	SI unit	Reference
Lattice parameter	Planck length	l_P	$1.61624(12) \cdot 10^{-35}$	m	[6]
Poisson ratio		ν	0.25	–	[7]
Mass of particle	Planck mass	m_P	$2.17645(16) \cdot 10^{-8}$	kg	[6]
Frequency of the internal process	Inverse of Planck time	$f_P = 1/t_P$	$1.85486(98) \cdot 10^{43}$	s^{-1}	[6]
Lamé constants	Energy density	$\lambda = \mu_L$	$1.853240963 \cdot 10^{114}$	kg m^{-1} s^{-2}	This work
Planck potential	Square of velocity	$\mu_0 = kT_P/m_P$	$8.987534456 \cdot 10^{16}$	M^2 s^{-2}	This work
Standard mechanical potential	Square of velocity	$\mu_0^m = \mu_0/2$	$4.493767228 \cdot 10^{16}$	M^2 s^{-2}	This work
Number of particles in unit cell			4		This work
Planck mobility	Duration of the jump	B_P	$5.391203018 \cdot 10^{-44}$	s	This work
Temperature	Planck temperature	T_P	$1.41679(11) \cdot 10^{32}$	K	[6]

interstitial Planck particles to lattice sites. The self-diffusivities of the Planck mass, D^*, and of interstitial defects, D_P^*, are related by $D^* = (\rho_P/\rho)D_P^*$. From the Nernst-Einstein relation ($D^* = BkT$) [8], the analogous relation holds for the mobilities:

$$B = (\rho_P/\rho)B_P, \tag{1}$$

where B_P and B denote the mobilities of the defects and Planck particles, respectively.

The collective movement ("the *particle*-like" behaviour) is common in fluids (e. g., vortex, soliton) and solids (e. g., complex defects and standing waves [9]). The *particle* (i. e., body) is equivalent to such a wave and is characterized by its energy, $Mc^2 = 4m_P l_P^{-3} \int_{-\infty}^{\infty} e \mathrm{d}v$, were M is the mass that is attributed to the *particle* and e denotes the total energy density in the PKC. If the *particle* is treated as complex, then by analogy, its mobility, B_M , equals

$$B_M = (m_P/M)B_P. \tag{2}$$

The drift velocity concept is used to define fluxes [10, 11]. The local (υ_i), diffusional (υ_i^d) and common drift velocities (drift due to deformation, $\upsilon^{drift} = \upsilon^\sigma$) in PKC are related by

$$\upsilon_i := \upsilon_i(t,x) = \upsilon_i^d(t,x) + \upsilon^\sigma(t,x), \quad \text{where } \upsilon^\sigma = \frac{\partial \boldsymbol{u}}{\partial t}. \tag{3}$$

$\boldsymbol{u}$ is the displacement vector. The diffusion fluxes, $J_i^d(t,x)$, are given by the Nernst-Planck formula [12]

$$J_i^d = \rho_i \upsilon_i^d = \rho_i B_i F = \rho_i B_i \,\mathrm{grad}\, \mu^m, \tag{4}$$

where i denotes the Planck particle (P_i) or complex (M), F and μ^m are the force and mechanical potential.

The calorimetric equation of state implies that $U = U(S,V,m)$. When a fixed volume is considered: $U/V = \check{\mathcal{E}} = \check{\mathcal{E}}(\check{s}, \rho_m)$ or $\rho_m \varepsilon = \check{\mathcal{E}}(\rho_m s, \rho_m)$, where $\check{\mathcal{E}}$ and $\check{s}$ denote the internal energy and entropy expressed as the energy per unit volume, ε and s are expressed per mass unit, $\rho_m = \rho_m(t,x) = m_P \rho$ is the density of mass, and m_P is the Planck mass. Thus, the Gibbs equation becomes [13]

$$\begin{aligned} &\mathrm{d}(\rho_m \varepsilon) = T\mathrm{d}(\rho_m s) + \mu^m \mathrm{d}\rho_m \\ &\text{or } \mathrm{d}(\rho\varepsilon) = T\mathrm{d}(\rho s) + \mu^m \mathrm{d}\rho, \end{aligned} \tag{5}$$

where $\mu^m = \partial(\varepsilon\rho_m)/\partial\rho_m$ denotes the mechanical potential.

Equation (5) is used to compute the rate of entropy production as a result of defect formation and diffusion [14]. The formation of defects and their diffusion does not affect the energy density. Both processes decrease the thermal ($Ts\rho$) and increase the mechanical ($\mu^m\rho$) energy and are represented by the last terms in (11) and (12).

When the deformation is small, the mechanical potential depends on the total energy density:

$$\begin{aligned} &\mu^m = \mu_0^m + \mu_0^m \ln(e/\mu_0^m) \\ &\text{or } e = \mu_0^m \exp\left((\mu^m - \mu_0^m)/\mu_0^m\right) \\ &\quad = \frac{\mu_0}{2} \exp\left(2(\mu^m - \mu_0^m)/\mu_0\right), \end{aligned} \tag{6}$$

where μ_0 and μ_0^m are the Planck and standard mechanical potentials, see Table 1. To distinguish the energy of the local deformation field, μ_{Re}^m, and the energy of the "internal process", μ_{Im}^m, the following notation is used:

$$\mu^m = \mu_0^m + \mu_{Re}^m - \mathrm{i}\mu_{Im}^m. \tag{7}$$

Combining (6) and (7) and the relation between the potentials (Table 1), the energy density is expressed by

$$e = \frac{\mu_0}{2} \exp\left(2(\mu_{Im}^m - \mathrm{i}\mu_{Im}^m)/\mu_0\right). \tag{8}$$

2.2. Mass, Momentum and Energy Balances

These are described by classical formulae with rigorous use of the caloric equation of state. To simplify and shorten the problem, the assumptions of a flat temperature field in the crystal (negligible heat flux, $J_q = 0$) and negligible viscosity are introduced. Consequently, the conservation of mass (particles), momentum and energy are given by

$$\partial\rho/\partial t = -\operatorname{div}(\rho\upsilon) = -\operatorname{div}(\rho\upsilon^{\sigma} + \rho\upsilon^{\mathrm{d}}), \quad (9)$$

$$\rho\partial\upsilon/\partial t = \rho\upsilon \operatorname{grad} \upsilon + \operatorname{div} \sigma, \quad (10)$$

$$\begin{aligned}\partial(\mu^{\mathrm{m}}\rho)/\partial t = &-\operatorname{div}(\mu^{\mathrm{m}}\rho\upsilon) + \sigma : \operatorname{grad}\upsilon \\ &+ \rho\upsilon^{\mathrm{d}}\operatorname{grad}\mu + \gamma\rho e,\end{aligned} \quad (11)$$

$$\partial(Ts\rho)/\partial t = -\operatorname{div}(Ts\rho\upsilon) - \rho\upsilon^{\mathrm{d}}\operatorname{grad}\mu^{\mathrm{m}} - \gamma\rho e, \quad (12)$$

$$\partial(\rho e)/\partial t = \operatorname{div}(\sigma\otimes\upsilon - \rho e\upsilon), \quad (13)$$

where $e = \varepsilon + 1/2\upsilon^2 = Ts + \mu^{\mathrm{m}} + 1/2\upsilon^2$ is the energy density (sum of the internal and kinetic energy) and $\sigma = \sigma'/m_{\mathrm{P}}$. In the thermal, (12), and mechanical parts, (11), of the total energy balance, (13), the terms describing the Frenkel defect formation, $\pm\gamma\rho e$, and diffusion, $\pm\rho\upsilon^{\mathrm{d}}\operatorname{grad}\mu^{\mathrm{m}}$, were introduced [14].

2.3. Mechanical Properties

The stress tensor of an ideal regular crystal is given by $\sigma = \sigma'/m_{\mathrm{P}} = (\lambda/m_{\mathrm{P}}\mathbf{trD})\mathbf{1} + 2\mu_{\mathrm{L}}/m_{\mathrm{P}}\mathbf{D} \overset{\lambda=\mu_{\mathrm{L}}}{=} (\lambda/m_{\mathrm{P}}\mathbf{trD})\mathbf{1} + 2\lambda/m_{\mathrm{P}}\mathbf{D}$, where $\mathbf{D}$ denotes the deformation tensor (the symmetrical part of the strain tensor) and $\lambda = \mu_{\mathrm{L}}$ are the Lamé coefficients of an ideal regular crystal.

It was shown by Cauchy and Saint Venant that, if the particles composing a regular crystal interact pairwise through central forces, then there is an additional symmetry requiring $C_{44} = C_{12}$ [15]. This implies the Poisson ratio 0.25 and $\mu_{\mathrm{L}} = \lambda$ [15]. Using the identity: $\operatorname{grad}\operatorname{div}\boldsymbol{u} = \operatorname{div}\operatorname{grad}\boldsymbol{u} + \operatorname{rot}\operatorname{rot}\boldsymbol{u}$, we have

$$\begin{aligned}\operatorname{div}\sigma &= 2\lambda/m_{\mathrm{P}}\operatorname{grad}\operatorname{div}\boldsymbol{u} + \lambda/m_{\mathrm{P}}\operatorname{div}\operatorname{grad}\boldsymbol{u} \\ &\equiv 3\lambda/m_{\mathrm{P}}\operatorname{div}\operatorname{grad}\boldsymbol{u} + \lambda/m_{\mathrm{P}}\operatorname{rot}\operatorname{rot}\boldsymbol{u}.\end{aligned} \quad (14)$$

The Young modulus and the ratio of the longitudinal to the transverse wave velocities equal $(c_{\mathrm{L}}/c_{\mathrm{T}})^2 = 3$ and $E = 2.5\lambda$ [15].

Only the isothermal processes are analyzed, consequently $f_{\mathrm{P}}, T_{\mathrm{P}}, B_{\mathrm{P}}, \lambda, \gamma =$ const. Upon introducing the mass diffusion flux, (3) and (4), and stress tensor, (14), formulae (9) – (13) become

$$\partial\rho/\partial t = \operatorname{div}(\rho N_{\mathrm{P}}B_{\mathrm{P}}\operatorname{grad}\mu^{\mathrm{m}}) - \operatorname{div}(\rho\upsilon^{\sigma}), \quad (15)$$

$$\rho \mathrm{D}\upsilon/\mathrm{D}t|_{\upsilon} = 3\frac{\lambda}{m_{\mathrm{P}}}\operatorname{div}\operatorname{grad}\boldsymbol{u} + \frac{\lambda}{m_{\mathrm{P}}}\operatorname{rot}\operatorname{rot}\boldsymbol{u}, \quad (16)$$

$$\begin{aligned}\partial(\mu^{\mathrm{m}}\rho)/\partial t = &-\operatorname{div}(\mu^{\mathrm{m}}\rho\upsilon) + \sigma : \operatorname{Grad}\upsilon \\ &+ \rho\upsilon^{\mathrm{d}}\operatorname{grad}\mu^{\mathrm{m}} + \gamma\rho e,\end{aligned} \quad (17)$$

$$\partial(\rho e)/\partial t = \operatorname{div}(\sigma\otimes\upsilon - \rho e\upsilon). \quad (18)$$

The unknowns are ρ, $\boldsymbol{u}$, μ^{m} and e. Below, only uncoupled, elementary processes will be analyzed.

2.4. Newtonian Constant of Gravitation

The mechanical energy conservation, (17), allows us to estimate the gravity field generated by the immobile body, mass M, in the PKC. Gravity is created by diffusing Planck particles. The immobile body implies a quasi-stationary situation and the deformation, mass and energy in a space occupied by the body are "fixed": $\mu^{\mathrm{m}} = \mu^{\mathrm{m}}(x)$, $\rho = \rho(x)$ and $\upsilon^{\sigma} = \partial\boldsymbol{u}/\partial t \cong 0$. One can assume: $\sigma : \operatorname{Grad}\upsilon \cong 0$. Thus, combining (3) and (17):

$$\mu^{\mathrm{m}}\operatorname{div}(\rho_{\mathrm{P}}\upsilon^{\mathrm{d}}_{\mathrm{P}}) = \gamma\rho e. \quad (19)$$

The diffusion velocity depends on the real part of the mechanical potential $\mu^{\mathrm{m}} \cong \mu^{\mathrm{m}}_{\mathrm{Re}}$, and becomes

$$\upsilon^{\mathrm{d}}_{\mathrm{P}} = -B_{\mathrm{P}}\operatorname{grad}\mu^{\mathrm{m}}_{\mathrm{Re}}(t,x). \quad (20)$$

The mass-energy equivalence in the PKC, $m_{\mathrm{P}}\rho e = c^2\rho_{\mathrm{M}}$ ($E = Mc^2$), implies:

$$e = c^2\rho_{\mathrm{M}}/(m_{\mathrm{P}}\rho). \quad (21)$$

Assuming that the defect concentration does not vary markedly, $\rho_{\mathrm{P}} \cong$ const., and combining (19) – (21):

$$\operatorname{div}\operatorname{grad}\mu^{\mathrm{m}}_{\mathrm{Re}} \cong -\rho_{\mathrm{M}}(\gamma c^2)/(m_{\mathrm{P}}\mu^{\mathrm{m}}B_{\mathrm{P}}\rho_{\mathrm{P}}). \quad (22)$$

When the local energy density due to the mass M does not change markedly, the average value of the mechanical potential in the PKC can be used: $\mu^{\mathrm{m}} \cong \bar{\mu}^{\mathrm{m}} = 2\mu^{\mathrm{m}}_0/\pi = \mu_0/\pi$. The density of defects and the frequency of their formation are given by $\rho_{\mathrm{P}} \cong \alpha l_{\mathrm{P}}^{-3}\exp(-E_f/kT_{\mathrm{P}})$ and $\gamma \cong f_{\mathrm{P}}\alpha\exp(-E_f/kT_{\mathrm{P}})$, where α and f_{P} are the geometrical factor and the

Table 2. The computed physical constants of the Planck – Kleinert Crystal.

Physical Quantity	Unit	Symbol for unit and relations with other units	Value in SI units	SI unit	Reference
Planck density	Mass density	$4m_\mathrm{P}l_\mathrm{P}^{-3}$	$2.062008662 \cdot 10^{97}$	$\mathrm{kg\,m^{-3}}$	This work
Gravitational constant		$G = c^2 l_\mathrm{P}/m_\mathrm{P}$	$6.67418999 \cdot 10^{-11}$ $6.6742(10) \cdot 10^{-11}$	$\mathrm{m^3\,kg^{-1}\,s^{-2}}$	This work [6]
Self-diffusion coefficient	Planck diffusivity	D_P^*	$4.845362288 \cdot 10^{-27}$	$\mathrm{m^2\,s^{-1}}$	This work
Planck energy	Energy	$E_0 = l_\mathrm{P}^3\lambda/4$	$1.9560971 \cdot 10^9$ $1.9560932 \cdot 10^9$	$\mathrm{kg\,m^2\,s^{-2}}$	This work [6]
Planck constant	(Particle energy in one cycle) · 4π	$h = 2\pi m_\mathrm{P} B_\mathrm{P} c^2$	$6.626069311 \cdot 10^{-34}$ $6.6260693(11) \cdot 10^{-34}$	$\mathrm{kg\,m^2\,s^{-1}}$	This work [6]
Speed of transverse wave	Velocity	$c = \sqrt{l_\mathrm{P}^3\lambda/4m_\mathrm{P}}$	299 792 458 299 792 458	$\mathrm{m\,s^{-1}}$	This work [6]
Speed of longitudinal wave	Velocity	$c_\mathrm{L} = \sqrt{3}\,c$	519 255 769	$\mathrm{m\,s^{-1}}$	This work

Planck frequency [16]. Upon substituting the above relations and data from Tables 1 and 2, (22) becomes

$$\begin{aligned}\operatorname{div}\,\operatorname{grad}\mu_\mathrm{Re}^\mathrm{m} &= -4\pi\rho_\mathrm{M}(l_\mathrm{P}^3 f_\mathrm{P} c^2)/(m_\mathrm{P}\mu_0 B_\mathrm{P})\\ &= -4\pi\rho_\mathrm{M}(l_\mathrm{P} c^2)/m_\mathrm{P}\\ &= -4\pi G\rho_\mathrm{M},\end{aligned}\tag{23}$$

which is the equation discovered by the French mathematician Siméon-Denis Poisson, and $G = 6.674189 \cdot 10^{-11}$ is the Newtonian constant of gravitation (NIST data: $G = 6.6742(10) \cdot 10^{-11}$ [7]). It was shown that the diffusing interstitial Planck particles are equivalent to weakly interactive massive particles and create the gravitational interaction between matter. One can consider the analogies between the "dark matter" and diffusing Planck particles as well as between the "dark energy" and energy of the diffusing Planck particles.

2.5. The Time-Dependent Schrödinger Equation

The deformation and its velocity as well as diffusion of defects are now assumed to be negligible, $\upsilon^\sigma = \upsilon_\mathrm{P}^\mathrm{d} = 0$ and $\operatorname{div}(\sigma \otimes \upsilon) \cong 0$. The process that governs the de Broglie waves is the fast internal process. We analyze the case when the driving force of the transport (the collective Planck mass movement, i. e., the movement of a complex of particles showing an energy E and mass $M = Ec^{-2}$) is controlled by the imaginary part of the mechanical potential, $\mu^\mathrm{m} = \mu_\mathrm{Im}^\mathrm{m}$. The flux given by (4) equals $J_\mathrm{M}^\mathrm{d} = -\rho_\mathrm{M} B_\mathrm{M}\operatorname{grad}\mu_\mathrm{Im}^\mathrm{m} = -\rho e B_\mathrm{M}\operatorname{grad}\mu_\mathrm{Im}^\mathrm{m}$, and (18) becomes

$$\partial(\rho e)/\partial t = B_\mathrm{M}\operatorname{div}(\rho e\operatorname{grad}\mu_\mathrm{Im}^\mathrm{m}).\tag{24}$$

By combining (2), (8) and (24) one gets

$$\begin{aligned}&\partial\exp(2(\mu_\mathrm{Re}^\mathrm{m} - \mathrm{i}\mu_\mathrm{Im}^\mathrm{m})/c^2)/\partial t =\\ &(m_\mathrm{P}B_\mathrm{P})/M\operatorname{div}(\exp(2(\mu_\mathrm{Re}^\mathrm{m} - \mathrm{i}\mu_\mathrm{Im}^\mathrm{m})/c^2)\operatorname{grad}\mu_\mathrm{Im}^\mathrm{m}).\end{aligned}\tag{25}$$

By denoting $\psi = \exp((\mu_\mathrm{Re}^\mathrm{m} - \mathrm{i}\mu_\mathrm{Im}^\mathrm{m})/c^2)$, $E = m_\mathrm{P}/(2Mc^2)B_\mathrm{P}((\nabla\mu_\mathrm{Im}^\mathrm{m})^2 + c^2\Delta\mu_\mathrm{Re}^\mathrm{m} + (\nabla\mu_\mathrm{Re}^\mathrm{m})^2)$ and using the identity $\psi^{-1}\Delta\psi = \Delta(\mu_\mathrm{Re}^\mathrm{m} - \mathrm{i}\mu_\mathrm{Im}^\mathrm{m})/c^2 + (\nabla(\mu_\mathrm{Re}^\mathrm{m} - \mathrm{i}\mu_\mathrm{Im}^\mathrm{m})/c^2)^2$, (25) becomes the standard Schrödinger equation:

$$\begin{aligned}\mathrm{i}\partial\psi/\partial t &= -(m_\mathrm{P}B_\mathrm{P}c^2)/(2M)\operatorname{div}\,\operatorname{grad}\psi + E\psi\\ &= -h/(4\pi M)\operatorname{div}\,\operatorname{grad}\psi + E\psi.\end{aligned}\tag{26}$$

Using data shown in Table 1 one can compute the value of Planck's constant, Table 2.

2.6. Transverse Waves in Planck – Kleinert Crystal

The wave propagation can be analyzed in two limiting situations, the longitudinal wave: $\operatorname{rot}\boldsymbol{u} = 0$, and the transverse one: $\operatorname{div}\,\operatorname{grad}\boldsymbol{u} = 0$. When mass transport occurs solely due to deformation ($\upsilon = \upsilon^\sigma$), the deformation is low ($\rho \cong$ const.) and the longitudinal wave is excluded, (16) becomes the equation of a transverse wave with the velocity given by $c = \sqrt{l_\mathrm{P}^3\lambda/4m_\mathrm{P}}$:

$$\partial\upsilon^\sigma/\partial t = \partial^2\boldsymbol{u}/\partial t^2 = \lambda l_\mathrm{P}^3/(4m_\mathrm{P})\operatorname{rot}\,\operatorname{rot}\boldsymbol{u}.\tag{27}$$

2.7. Maxwell Equations in Vacuum

Suppose that $\boldsymbol{E} = -\operatorname{rot}\boldsymbol{u}$, $\boldsymbol{B} = \upsilon^\sigma$ and the permeability equals $\mu_0^B = 4m_\mathrm{P}/l_\mathrm{P}^3\lambda = \rho_\mathrm{m}/\lambda$. Then, (27) becomes

$$\mu_0^B\partial\boldsymbol{B}/\partial t = -\operatorname{rot}\boldsymbol{E}.\tag{28}$$

The second equation follows straightforwardly. Differentiation of the electric field strength, $\boldsymbol{E} = -\operatorname{rot}\boldsymbol{u}$, yields $\partial\boldsymbol{E}/\partial t = -\partial\operatorname{rot}\boldsymbol{u}/\partial t = -\operatorname{rot}\partial\boldsymbol{u}/\partial t$, which by substituting $\boldsymbol{B} = \upsilon^{\sigma} = \partial\boldsymbol{u}/\partial t$ results in

$$\varepsilon_0\partial\boldsymbol{E}/\partial t = -\operatorname{rot}\boldsymbol{B}. \tag{29}$$

The permittivity, ε_0, equals one, and $\varepsilon_0\mu_0^B = c^{-2}$. The other relations that form the Maxwell system are the consequence of an "*empty lattice*" and low deformation. From (15), the usual formula for noncompressible flow follows: $\operatorname{div}\upsilon^{\sigma} = 0$. Thus, from the assumption of low deformation:

$$\operatorname{div}\boldsymbol{B} = \operatorname{div}\upsilon^{\sigma} = 0. \tag{30}$$

The *charged particles* are not included in the present analysis. Upon differentiating the constitutive equation for $\boldsymbol{E}$ one gets: $\operatorname{div}\boldsymbol{E} = -\operatorname{div}\operatorname{rot}\boldsymbol{u}$. Consequently, from the identity $\operatorname{div}\operatorname{rot}\boldsymbol{u} = 0$, it follows that

$$\operatorname{div}\boldsymbol{E} = 0. \tag{31}$$

Equations (28) – (31) form the Maxwell system in *vacuum*.

3. Summary

The results support the physical reality at the Planck scale and allow for the interpretation of gravity, quantum mechanics and electromagnetic phenomena. Faster than light velocities of the longitudinal wave might be considered as unphysical, and this requires "fine-tuning" or higher symmetry to make all "sound speeds" equal. Here, the different velocities are related to specific force fields and were analyzed as real quantities that mark different time scales. The diffusing interstitial Planck particles (defects) create the gravitational interaction between matter. The collective behaviour of the particles forming the Planck-Kleinert Crystal is equivalent to the *particle* (*body*). Such an approach enables the Schrödinger equation to be derived. The transverse wave in the PKC is equivalent to an electromagnetic wave in vacuum. The PKC model allows the derivation of the classical Newton's law of gravity (the Poisson equation). The consequence of the equation of internal energy conservation is the existence of waves involving temperature, but not the mechanical potential variations. They are analogous to "the second sound" described by Landau and Lifschitz [14].

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[1] F. Wilczek, Phys. Today **52**, 11 (1999).
[2] J. L. Lebowitz, Rev. Modern Phys. **71**, S347 (1999).
[3] M. Bojowald, Nature **436**, 920 (2005).
[4] H. Kleinert, Ann. Phys. **44**, 117 (1987).
[5] H. Kleinert and J. Zaanen, Phys. Lett. A **324**, 361 (2004).
[6] National Institute of Standards and Technology, http://physics.nist.gov (2006).
[7] R. S. Lakes, Elastic Freedom in Cellular Solids and Composite Materials, in: Mathematics of Multiscale Materials, IMA Vol. 99 (Eds. K. Golden et al.), Springer, Berlin 1998, pp. 129.
[8] A. Einstein, Ann. Phys. **17**, 549 (1905).
[9] J. Giannoulis and A. Mielke, Nonlinearity **17**, 551 (2004).
[10] L. S. Darken, Trans. AIME **174**, 184 (1948).
[11] K. Holly and M. Danielewski, Phys. Rev. B **50**, 13336 (1994).
[12] M. Planck, Ann. Phys. Chem. **40**, 561 (1890).
[13] J. W. Gibbs, Trans. Conn. Acad. **2**, 382 (1873)
[14] L. D. Landau and E. M. Lifschitz, Fluid Mechanics, 2nd ed., Butterworth-Heinemann, Oxford 1987.
[15] M. P. Marder, Condensed Matter Physics, John Wiley & Sons, New York 2000, pp. 287-303.
[16] G. E. Murch, Atomic Diffusion Theory in Highly Defective Solids, Trans Tech House, Aedermannsdorf 1980.